November 20, 2024

Handout for Week 11

Dissecting Reason Relations with a Substitutional Scalpel

Outline:

Recap: The argument of "What are Singular Terms, and Why are There any?"
Why substitution-structural syntactic roles and substitution-inferential significances must line up as they do: otherwise substitutional codification of inferential patterns is incompatible with the expressive power of conditionals and negation at the sentential level.

The current project: Decompose sentences substitutionally, to discern *semantically significant subsentential structure*, even in the absence of perspicuous syntactic subsentential components.
Examples: Dolphins and PDP-nets (where an asserted sentence might be a set of weights/activations).
Relevant prior efforts: combinatory logic (Quine and Schönfinkel), van Fraassen "Quantification as an Act of Mind," Brandom "Singular Terms and Sentential Sign Designs."

Suggestive discovery: Articulating substitutional relations functionally, top-down, reveals a structural halfway point between the full substitutional structure of terms and complex predicates (as elaborated in WASTWATA) and no semantically significant subsentential substitutional structure. This *semisubstitutional* structure, in which proto-terms and proto-predicates are *uncorrelated* distinguishes simple proto-predicates as substitutional equivalence-classes of sentences in which they occur, and assigns to each sentence the set of proto-terms occurring in it. But complex predicates are not discernible, since there is nothing corresponding to a term occurring in a sentence at one argument-place rather than another of the simple predicate.

The Construction (in 7 steps): We put conditions on a set of implication-space models M defined over a language L being a term/predicate *dissecting model-set*: one that permits the imputation of full first-order subsentential term/predicate structure. Different dissecting model-sets define different decompositions.

1) Define the set of intersubstitution-pairs of sentences A of L endorsed by each model m in M.
   - Use CO-pairs to define for each model the set $Id_L(m)$ of pairs of sentences m treats as substitutional variants of one another.

For A,B$\in$L, $\mathbf{\{A,B\}\in Id_L(m)}$ iff $\forall S,S'\subseteq L$ $<S\cup\{A\},S'\cup\{B\}>\in \mathbf{I}_m$ and $<S\cup\{B\},S'\cup\{A\}>\in \mathbf{I}_m$.

2) Define *simple predicates* (sentence varieties) of L according to M as sets of mutually substitutionally variant sentences $|A|_M$.
   - $\mathbf{A_i \approx_M A}$ **iff** $\mathbf{\exists m\in M[\{A,A_i\}\in Id(m)]}$.

Stipulation 1: $\forall$**A,B,C**$\in$**L**$\exists$**m,m'**$\in$**M[{A,B}**$\in$**Id$_L$(m) & {B,C}**$\in$**Id$_L$(m')]** $\Rightarrow$ $\exists$**m"[{A,C}**$\in$**Id$_L$(m")]**.
Stipulation 1 (the transitive closure of Id_L-sets) ensures that $\approx_M$ is an equivalence relation. So we can name the equivalence class of substitutional variants of A (according to M) as $|A|_M$.

- $A_i \in |A|_M$ **iff** $A_i \approx_M A$. Reflexivity holds by CO in def. of Id(m), symmetry by definition in terms of unordered pairs of sets of sentences (so $A_i \in |A|_M$ iff $A \in |A_i|_M$), and transitivity by Stipulation 1.
- (Simple) predicates are equivalence classes of sentences under this relation.

$\mathbf{Preds_M(L) = \{|A|_M : A \in L\}}$.

3) Define singular *terms* of L, as sets of sentences in which those terms "occur".
This is more complicated:
   i) Define M(A) for $A \in L$, by $\mathbf{M(A) = \{m \in M: \text{for some } B \neq A, \{A,B\} \in Id(m)\}}$.

M(A) is the set of models that implicitly CO-endorse nontrivial substitutions affecting A.
   ii) Define an ordering on sentences $A \leq_M B$ iff $M(A) \subseteq M(B)$. Every model that underwrites nontrivial substitutions affecting A underwrites nontrivial substitutions affecting B.
   iii) A is *minimal-monadic* in the $\leq_M$ ordering, $\mathbf{MinMo_{\leq M}(A) \text{ iff } B \leq_M A \Rightarrow A \leq_M B}$.
   iv) The set of *terms* of L according to M is

$\mathbf{T_M(L) = \{S \subseteq L: \exists A \in L[MinMo_{\leq M}(A) \,\&\, S = \{X \in L: A \leq_M X\}\,]\,\}}$.
This is the set of upward cones (in the $\leq_M$-ordering) above minimal-monadic sentences, which exhibit only one term: the set of all sentences that also exhibit that term (and usually also further terms).

4) Associate with each sentence the set of terms "occurring in" it.
   - The set of terms in A, $\mathbf{T_M(A) = \{S \in T_M(L): A \in S\}}$.

The terms are just sets of sentences. For the term to "occur in" the sentence is just for the sentence to be in—an element of—the set of sentences $T_M(A)$, which are the terms of A.
   - The number of terms in a sentence, $\mathbf{TNum_M(A) = card(T_M(A))}$ is just the cardinality of, the number of terms in $T_M(A)$, the term-set of A (according to M).

5) Associate with each simple predicate its adicity.
   - The *adicity* of a simple predicate—the number of "slots" it has to fit terms "into"—is the number of terms in the *most term-diverse variants* of that sentence variety: one that has the most distinct terms in it. We could just say that $\mathbf{Adic(|A|_M) = max(TNum_M(A_i))}$**, for** $\mathbf{A_i \in |A|_M}$.

These conditions define the ***semi-substitutional*****,** minimal semantically significant subsentential structure, which is weaker than the full term/predicate structure. Generally, we haven't put enough conditions on the sets of sentence-pairs $Id_L(m)$ to ensure that they are all and only term-substitutional variants of one another in the Guiding Interpretation.

Q1: What does a world look like, in which instead of particulars and properties of and relations among them, it consists of whatever these proto-terms and proto-properties-and-relations pick out?
Q2: What more is needed for full term/predicate structure?
A2: The semi-substitutional structure identifies simple predicates with sets of sentences, and terms with sets of sentences. These two sets of sets of sentences cross-classify them, in that every sentence is in exactly one simple-predicate set of sentences and may be in multiple term-sets of sentences. Full term/predicate substitutional structure requires a much more intricate kind of *correlation* between the terms and the predicates (cf. Van Fraassen).
Q3: How can we describe functionally the crucial job being performed by thinking of simple predicates as applying to *ordered* tuples of terms? How should we think metaphysically about this *ordering* in the states of affairs that are truth-makers and falsity-makers of essentially *a*symmetric relations such as "*x* admires *y*"? Cf. Kit Fine "Neutral Relations".

6) Impute classical term/predicate structure by defining, for each n-adic simple predicate $|A|_M$, the n *argument-functions* associated with it. Each argument function takes as arguments any sentence variants $A_i \in |A|_M$, and yields as value some single term $t \in T_M(A_i)$, the term occurring in Ai that occurs "at" or "in" the "argument place" defined by that function. The argument functions are indexed by the terms that occur "in" them in an arbitrarily chosen maximally term-diverse variant A whose term-set $T_M(A)$ is *disjoint* from the terms set $T_M(A_i)$ of A_i.
Strategy: In three stages:

i) Derive from $Id_L(m)$, the set of *sentence*-pairs model $m \in M$ treats as substitutional variants (by treating them functionally as CO-pairs) the set $Id_T(m)$ of *term*-pairs that m implicitly treats as related by true identities.
ii) Make the crucial *correlational stipulation* in terms of the $Id_T(m)$ and $Id_L(m)$ sets of *all* the models in the dissecting model-set M.
iii) Use the Id_T-sets to define argument-functions for every sentence-variety in L.

For (i):
TPM 1) $M' \subseteq M$ is the set of *singleton term-pair models* iff
$\forall m \in M' \forall X,Y \in L[(X \neq Y \ \&\ \{X,Y\} \in Id_L(m)) \Rightarrow (t \in T_L(X)\ \&\ t' \in T_L(Y)\ \&\ T_L(X)-\{t\}=T_L(Y)-\{t'\})]$.
The first two conjuncts of the consequent say that all the nontrivial sentence-variant-pairs in every singleton term-pair model differ in one exhibiting one of {t,t'} and the other sentence variant exhibiting the other. The final conjunct says that there are no *other* differences (in addition to t,t' differences) of the terms in paired sentence variants.
Then we can say that the *term-pair set* of model m, $\mathbf{Id_T(m) = \{\{t,t'\}\}}$, is the singleton.
For (ii), we work in two stages:

First: Stipulate that there is a full set of singleton term-pair models (one for every pair of terms). Every dissecting set of models M contains a subset M' of singleton term-pair models whose indices are all the pairs of terms in $T_M(L)$.
TPM 2) $\forall t,t' \in T_M(L) \exists m \in M'[Id_T(m) = \{\{t,t'\}\}]$.

Next: **the big *correlation stipulation*** is to require that every non-singleton term-pairs model in M can be assigned an Id_T-set that is the transitive closure of some some set of singleton *term* pairs, accordingly as its Id_L-set of *sentence* pairs can be factored as the transitive closure of the unions of the Id_L-sets of the singleton-pair models from which its Id_T-set is computed.

TPM 3) $\mathbf{\forall m \in M\ \exists M'' \subseteq M' [Id_T(m) = TrCl(\cup Id_T(m_i) : m_i \in M'')\ \&}$
$\mathbf{(Id_T(m) \Leftrightarrow (Id_L(m) = TrCl(\cup Id_L(m_j \in M'' : Id_T(m_j) \subseteq Id_T(m)))]}$.

The first clause says that every model in the dissecting set M has a set of term-pairs that is the transitive closure of the union of some set of singleton term-pairs, and the second clause says that the CO-*sentence* pair set $Id_L(m)$ is the transitive closure of the *sentence* pair sets of the singleton term-pair models whose singular term-pairs were unioned to get the STPs of the non-singleton term-pair models.

We can show that all these stipulations are jointly satisfiable (consistent) because the Guiding Interpretation gives us a model that satisfies them.

<u>Defining argument-functions</u> (which is (iii) above):

First, we pick a *maximally term diverse* variant $A' \in |A|_M$, that is, s.t. $TM(A') = Adic(|A|_M)$.

Use its n distinct terms to index the argument functions of $|A|_M$: $T_M(A') = \{t_1, \ldots t_j, \ldots t_n\}$, and we want to define n argument functions $f_{tj}(A_i)$.

$\mathbf{f_{tj}(A_i) = t \text{ iff } \forall m \in M\ [\{A', A_i\} \in Id_L(m) \Rightarrow \{t_j, t\} \in Id_T(m)]}$.

Every model that witnesses A_i and A' being sentence variants of one another (members of the same sentence variety) has a term-pair set that includes $\{t_j, t\}$.

7) Show that by the construction in (1)-(6) each sentence in every set of syntactically perspicuous sentences in the Guiding Interpretation gets mapped onto a sentence in L, and *vice versa*, and that the semantically significant subsentential structure defined by assigning each n-ary sentence-variety n argument-functions (accordingly) suffices for the introduction of logical vocabulary in the form of quantifiers and identity.

<u>Strategy</u>:

Show that argument-functions suffice to define argument *places* for these two purposes:

i) Associate with each sentence the set of all its sentence-variants that differ from it *only* at some *one* designated argument-place: $\mathbf{\lambda_{tj}(A_i) =_{df.} \{X \in |A_i|_M : \forall k \neq j [ftk(X) = ftk(A_i)]}$.

and

ii) Given any such set of sentence-variants that differ only at one argument-place, and any term, to compute the sentence-variant in which *that* term occurs in *that* argument-place: $\mathbf{\lambda_{tj}(A_i, t) =_{df.} \text{ the } X \in \lambda_{tj}(A_i) : f_{tj}(X) = t}$. Will need to show existence and uniqueness.

These correspond to the two fundamental operations of the λ-calculus:

Operation (i) is λ-*abstraction*, and operation (ii) is λ-*application*.

It is known that the whole first-order predicate calculus can be built up from repeated applications of these two operations.

Next: Introduce quantifiers and identity as logical locutions codifying predicate- and term-implications.